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COMPUTATION OF ATTACHED FLOW PAST AN AIRFOIL PROFILE AT HIGH REYNOLDS NUMBERS*

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A mathematical model of an attached flow of an incompressible fluid past an airfoil profile at high Reynolds numbers is proposed. The model enables one to determine the effect of viscosity on the magnitude of aerodynamic characteristics. Not only is the displacing action of the turbulent boundary layer and wake on the external flow taken into account, but the solution in the neighbourhood of the trailing edge is also studied, and this makes it possible to formulate a more accurate analogue of the Chaplygin-Zhukovskii condition. Comparison of numerical results with experimental data shows that the accuracy of the results is comparable with that of experiment. The flow past a profile is usually computed by solving a sequence of problems arising when the concept of the Prandtl boundary layer is applied regularly. In this approach the external problems describe the flow of an inviscid fluid past modifications of the profile, which take into account the displacement of the boundary layer and distortion of the wake. Their unique solution satisfies the additional demand of regularity. To a first approximation such a demand is represented by the Chaplygin-Zhukovskii condition. To a second approximation the condition is obtained by analysing the solution of the Navier-Stokes equations near the trailing edge of the profile. In the present paper the analysis is carried out for a profile with a sharp trailing edge, in which the angle between the tangents is not zero.

1. Let us consider the flow of an incompressible fluid past an airfoil profile. We shall regard the segment of the straight line between the leading and trailing edge of length L as the chord of the profile, and the angle α between the direction of the velocity at infinity U_∞ and the chord, as the angle of attack. We shall refer all linear parameters to L, the velocity to U_∞, and the pressure p to the square of the pressure head ρU_∞^2 where ρ is the density of the fluid. We shall place the origin of a Cartesian coordinate system xy at the trailing edge of the profile, and direct the x axis along the bisector of the angle β ($\beta \ll 1$) within it.

We shall consider the solution of the problem of the flow of an ideal incompressible fluid past a profile in the plane $\zeta = r\exp(i\omega)$. The outside of the profile will map onto the outside of the unit circle $|\zeta| = 1$ in this plane, and the trailing edge of the profile will correspond to the point $\zeta = 1$.

We can assume, without loss of generality, that such a mapping can be carried out using the method described in /1/. We shall use the Karman-Trefftz transformation

$$(\xi - \xi_0)/(\xi + \xi_0) = [(z - z_0 - k_1\xi_0)/(z - z_0 + k_1\xi_0)]^{1/k_1},\ k_1 = 2 - \beta/\pi$$

to map the profile onto the part of the plane bounded by an almost spherical contour. After this we will seek the coefficients A_j and B_j of the Theodorsen-Garrick transformation

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$$\xi = \zeta \exp\left[\sum_{j=0}^{\infty} (A_j + iB_j)\zeta^{-j}\right]$$

We can write the derivative of the complex potential in the neighbourhood of the point $z = 0$, with $\beta \ll 1$, in the form

$$\frac{d\Phi}{dz} = U_{00}\left[1 + \frac{\beta}{2\pi}(1 + \ln z) - 2ik_0 z^{1/2} + \ldots\right] \tag{1.1}$$

$$U_{00} = \mathrm{Re}\,\frac{2\exp(A_0)\cos(\alpha - B_0)}{k_1\xi_0(1 - M_1)^2}, \quad 2k_0 = \mathrm{Re}\,\frac{\mathrm{tg}(\alpha - B_0) - 3iM_2}{1 - M_1}$$

$$M_n = \sum_{j=1}^{\infty} j^n (A_j + iB_j), \quad n = 1, 2$$

The coefficient k_0, which is equal to the sum of two terms, one of which is proportional to the circulation and the other to the load at the trailing edge at zero lift, appears in the estimate of the curvature of the stream line emerging from the trailing edge: $k_0 x^{-1/2} + O(1)$.

2. Let us consider the solution of the boundary-layer equations near the trailing edge, on the upper surface of the profile, where the flow is retarded. We shall take U_{00}, as the unit velocity, and refer the coordinate y, the transverse component of velocity v and turbulent friction intensity τ to characteristic thickness δ of the boundary layer. Then the equations can be written in the form

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{dp}{dx} = \frac{\partial \tau}{\partial y}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

When $y = 0$ $u = v = 0$, while when $y \to \infty$ we obtain from (1.1), for small $x < 0$,

$$u = 1 + f(x), \quad p = p_{00} - f(x)$$
$$f(x) = 2k_0(-x)^{1/2} + (2\pi)^{-1}\beta(1 + \ln|x|) + \ldots \tag{2.2}$$

The order of magnitude of k_0 should be determined in accordance with the assumption that the flow is attached. In the case of laminar flow, the stream becomes detached in the region of the trailing edge when $k_0 = O(R^{-1/16})$ /2/ (R is the Reynolds number). Therefore, an attached laminar flow is possible only when $k_0 \ll 1$. When the flow separates from a smooth surface, the pressure distribution near the point of separation is given by the first two terms of (2.2), and in the laminar flow we again have $k_0 = O(R^{-1/16})$/3/.The experiment, however, indicates that the point of separation of a turbulent flow lies at an appreciable distance further downstream than the point of separation of the laminar boundary layer. This compels us to put $k_0 = O(1)$ in the case of turbulent separation from a smooth surface /4/, and assume that attached flows past a profile are possible when $k_0 = O(1)$.

We shall further assume that $k_0 = O(1)$, in which case the solution (2.1), (2.2) in the basic part of the boundary layer can be written in the form

$$u = U_0(y) + 2k_0(-x)^{1/2} u_{11}(y) + \beta u_{12}(y)\ln(-x) + \ldots \tag{2.3}$$
$$v = k_0(-x)^{-1/2} v_{11}(y) + \beta v_{12}(y)(-x)^{-1} + \ldots$$

The functions appearing in it are expressed in terms of $U_0(y)$. When y decreases, $U_0(y)$ also tends to zero and the repesentation (2.3) becomes invalid.

The flow in the neighbourhood of the trailing edge can be analysed in a natural way using asymptotic expansions in the small parameter δ. We find here that its form depends very much on the size of the boundary-layer region generating the principal term of the variation in the displacement thickness. In the laminar boundary layer the transverse dimension of this region is of the order of $O(\delta^{5/4})$ /5/. In the turbulent layer, where the mean velocity profile can be approximated, as $y \to 0$, by the power function

$$U_0(y) = by^{1/n}, \quad n > 2 \tag{2.4}$$

the interaction becomes supercritical /6/ and the change in the displacement thickness is determined by the main part of the boundary layer. This means that formulas (2.3) are sufficient for calculating δ^* as $x \to 0\_$

$$\frac{dV_e\delta^*}{dx} = \delta U_{00}\Lambda\left[\frac{k_0}{(-x)^{1/2}} - \frac{\beta}{2\pi x}\right], \quad \Lambda = \int_0^{\infty}(U_0^{-2} - 1)\,dy \tag{2.5}$$

The inclination of the stream lines in the boundary layer is of the order of $k_0\delta(-x)^{-1/2}$.

when $x = 0-$. On the other hand, the inclination of the stream lines in the outer potential flow at $x = 0+$ is given by the quantity $2k_0x^{1/3}$. Therefore, in the region of dimension $x = O(\delta)$, where these quantities are of the same order, the interaction between the outer flow and the boundary layer plays a decisive role. In order for the effect of rotating the flow by an angle β not to change the order of the inclination of the stream lines, we shall put $\beta = O(\delta^{1/3})$.

An entirely different situation occurs when the flow becomes detached in the region of the trailing edge of the profile. In this case the displacement thickness increases on approaching it logarithmically /4/ and the value of Λ calculated from (2.2) becomes appreciably greater than unity. Therefore the question of whether to use the boundary based on formulas (2.4) and (2.5) or the theory given in /4/, depends on the magnitude of Λ. Let us find Λ using the exerimental data given in /7/ for the velocity profiles in the cross-section of the wake passing through the trailing edge of the profile NACA 0012 obtained for the case of subsonic flow past it, at an angle of attack $3, 6, 9°$ and $R = 3.8 \cdot 10^5$. We will find that the corresponding values of Λ are 1, 1.5 and 5.2, and this means that formula (2.6) can be used.

3. The flow past a profile outside the boundary layer and the wake is described, also in the second approximation, by the solution of the Laplace equation in the z plane with a cut along the stream line emerging from the trailing edge of the profile. The effect of the displacing action of the boundary layer and wake, and of the curvature of the wake on the potential flow should be taken into account in such a manner as to satisfy the boundary conditions at the profile, as well as at the cut. They are given in /8/:

$$\frac{\partial \varphi_1}{\partial n} = \frac{dV_e\delta^*}{ds}, \quad \Delta\frac{\partial \varphi_1}{\partial n} = \frac{dV_e\delta^*}{ds}, \quad \Delta\frac{\partial \varphi_1}{\partial s} = -V_e\varkappa(s)(\delta^* + \theta) \tag{3.1}$$

The conditions for the jumps in the values of the functions denoted by Δ must hold at the cut, s and n are the arc length and the outer normal, $\varkappa(s)$ is the curvature of the stream line, V_e is the velocity at the outer boundary of the boundary layer, and θ is the thickness of the loss of momentum.

The solution of the boundary-value problem (3.1) is obtained using Cauchy-type integrals in the ζ plane where the circle corresponds to the contour of the profile. Here it is convenient to use the circle theorem /9/, according to which we must place on the line obtained by inverting the cut, sources of the same sign and vortices of opposite sign, and add to them the total vortex and source at the centre of the circle. The solution is obtained according to the Hilbert formula connecting the limit values of the real and imaginary parts of the analytic function. We will write an expression for correcting the tangential velocity component in the ζ plane

$$\begin{gathered}\frac{\partial \varphi_1}{\partial \omega} = \frac{1}{2\pi}\int_0^{2\pi} \frac{dV_e\delta^*}{d\omega'} \operatorname{ctg}\frac{\omega'-\omega}{2}\, d\omega' - \frac{i}{2\pi}\int_{O'D'} d(V_e\delta^*) + \\ \frac{1}{2\pi}\int_{O'D'} DV_e\varkappa(\delta^*+\theta)\, d\sigma - \frac{1}{2\pi}\exp(i\omega)\left[i\int_{O'D'} \frac{1-2\bar\tau\zeta+\bar\tau\tau}{(1-\bar\tau\zeta)(\tau-\zeta)}\frac{dV_e\delta^*}{d\sigma}\, d\sigma + \right. \\ \left.\int_{O'D'} DV_e\varkappa(\delta^*+\theta)\frac{1-\bar\tau\tau}{(1-\bar\tau\zeta)(\tau-\zeta)}\, d\sigma\right] + A_1\end{gathered} \tag{3.2}$$

Here ω is the angular coordinate of a point lying on the circle, τ is the complex coordinate of a point on the wake stream line $O'D'$, $d\sigma$ is the differential of the arc length and $D = |dz/d\zeta|$, A_1 is an arbitrary constant.

We shall investigate the behaviour of (3.2) as $z \to 0$ $(\omega \to 0)$. In the general case $(dV_e\delta^*/d\omega)_{\omega=0+} \neq (dV_e\delta^*/d\omega)_{\omega=0-}$ and $DV_e\varkappa(\delta^*+\theta) \neq 0$ at the point O'. This leads to the need to remove the logarithmic singularity from (3.2). Passing to the physical plane we obtain, taking (1.1) into account, an expression for correcting the longitudinal velocity component in the outer region, at $x < 0$, in the form

$$V_{1x}(x, 0+) = \delta(-x)^{-1/3}[A - \pi^{-1}k_0U_{00}h_0\ln(-x) + o(1)]$$

$$h_0 = \frac{1}{2}\int_{-\infty}^{+\infty}(U_0^{-2} - U_0^2)\, dy$$

The above expression shows that the solution of the external problem in the second approximation is, in general, not bounded. The constant A related to A_1 is calculated from the additional condition that the flow past the trailing edge of the profile is unbroken, and this condition is realized when the solution is constructed in the region of dimensions of the order of $O(\delta)$.

4. Let us now describe the flow in the δ-neighbourhood of the trailing edge of the profile. To do this we shall introduce the independent variables $x_2 = x/\delta,\ y_2 = y/\delta$ and put $\beta = \beta_0\delta^{1/2}$. The form of the expansions of the functions sought in small parameter δ is governed in this region by the relations (2.3):

$$U_{00}^{-1}V_x = U_0(y_2) + \delta^{1/2}u_2(x_2, y_2) - \beta \ln \delta u_{12}(y_2) + O(\delta) \tag{4.1}$$
$$U_{00}^{-1}V_y = \delta^{1/2}v_2(x_2, y_2) + O(\delta)$$
$$U_{00}^{-2}(p - p_{00}) = -(2\pi)^{-1}\beta(1 + \ln \delta) + \delta^{1/2}p_2(x_2, y_2) + O(\delta)$$

Substituting (4.1) into the equations of continuity and of projections of the momentum, we obtain a system of three equations in u_2, v_2 and p_2. respectively. Substituting the unknown function $v_2 = WU_0$ and eliminating u_2, we reduce this system to a single, linear, second-order partial differential equation

$$\frac{\partial^2 W}{\partial x_2^2} + U_0^{-2}\frac{\partial}{\partial y_2}\left(U_0^2\frac{\partial W}{\partial y_2}\right) = 0 \tag{4.2}$$

and a relation for the pressure

$$\partial p_2/\partial x_2 = U_0^2 \partial W/\partial y_2 \tag{4.3}$$

The solution of Eq.(4.2) satisfies the condition of zero flow at the boundaries of the cut along the negative half-axis $y_2 = 0,\ x_2 < 0$

$$W(x_2, 0\pm) = \mp\beta_0/2 \tag{4.4}$$

and the condition of increase at infinity, which follows from matching with (1.1)

$$p_2 + iW \to 2ik_0 z_2^{1/2} - (2\pi)^{-1}\beta_0 \ln z_2, \quad z_2 = x_2 + iy_2 \to \infty \tag{4.5}$$

Conditions (4.4) and (4.5) are sufficient to construct a unique solution of the boundary value problem only in the class of functions bounded when $z_2 \to 0$. Such a choice of the class of functions is equivalent to the requirement that a stream line emerge from the trailing edge. Within the scale of the thickness of the boundary layer, satisfying this requirement is less natural than the Chaplygin-Zhukovskii condition, but it enables us to ignore the neighbourhood of the trailing edge of the profile where the inertial terms are of the same order as the turbulent transfer terms. This simplifies the problem appreciably, although the assumption made must be checked by comparing the results obtained with experimental data.

In /10, 11/, where an analogous problem was discussed, additional assumptions were made concerning the order of deviation of $U_0(y_2)$ from unity. This made is possible to construct a solution of problem (4.2)-(4.5) in the form of an expansion in a small parameter. However, in order to ensure the necessary accuracy, they had to take three terms of this expansion. No such assumptions are made in the present paper, and problem (4.2)-(4.5) is solved exactly using the representation in the form of a Fourier integral

$$W = \frac{1}{2\pi}\int_{-\infty}^{+\infty} V(y_2)\exp(-i\lambda x_2)\,d\lambda \tag{4.6}$$

Substituting (4.6) into (4.2), we obtain a second-order ordinary differential equation for $V(y_2)$

$$U_0^{-2}\frac{d}{dy_2}\left(U_0^2\frac{dV}{dy_2}\right) - \lambda^2 V = 0 \tag{4.7}$$

Its solutions, bounded as $y_2 \to \pm\infty$, satisfy the conditions

$$dV/dy_2 = -|\lambda| V \operatorname{sign} y_2, \quad y_2 \to \pm\infty \tag{4.8}$$

In order to obtain the boundary conditions at $y_2 = 0\pm$, we shall consider the half-sum of the derivatives $\partial W/\partial y_2$ at $y_2 = 0\pm$. According to (4.4) this sum is equal to zero on the negative half-axis x_2. Therefore, its Fourier transformation represents the limit value, on the real axis $\operatorname{Im}\lambda = 0$, of the function analytic in the upper half-plane. Denoting this value by $\Phi^+(\lambda)$ we obtain, from (4.6),

$$\Phi^+(\lambda) = -i\lambda\,[V(0+) - V(0-)]/2$$

We determine the function $\Phi^-(\lambda)$ in exactly the same manner. This function represents the limit value, on the real axis, of the function analytic in the lower half-plane and is also the Fourier transform of the half-difference of $\partial p_2/\partial x_2$ at $y_2 = 0\pm$. The expression for it in terms of V and dV/dy_2 follows directly from relations (4.3) and (4.6). The third relation connecting the values of V at $y_2 = 0\pm$ is obtained from the Fourier transform of the difference $W(x_2, 0+) - W(x_2, 0-)$.

Let us further introduce the function

$$Q(y_2) = -\frac{U_0^2}{|\lambda|}\frac{dV}{V\,dy_2}\operatorname{sign} y_2$$

According to (4.7) the above function satisfies the equation

$$U_0^2 dQ/dy_2 = |\lambda|(Q^2 - U_0^4)\operatorname{sign} y_2 \tag{4.9}$$

The boundary conditions $Q(\pm\infty) = 1$ which follow from (4.8), enable us to construct integral curves of this equation at $y_2 < 0$ and $y_2 > 0$. In what follows, the limit values of $Q_1 = Q(0+, \lambda)$, $Q_2 = Q(0-, \lambda)$ will be of interest. The values are obtained by numerical integration of Eq.(4.9), and are bounded at all values of λ. The manner of their decay as $\lambda \to \infty$ is established /12/ using the method of matching the asymptotic expansions.

Let us now eliminate $V(0\pm)$ from the three equations listed above, containing the functions Φ^+, Φ^- and the Fourier transform of the difference $W(x_2, 0\pm)$. As a result we obtain the following linear relation:

$$\Phi^+(\lambda) = \frac{2i\operatorname{sign}\lambda}{Q_1 + Q_2}\Phi^-(\lambda) - \frac{\beta_0}{2}\cdot\frac{Q_1 - Q_2}{Q_1 + Q_2} \tag{4.10}$$

which represents an inhomogeneous problem of conjugation of the analytic function along the real axis. In the case of the flow past a profile in which the angle between the tangents at the trailing edge is zero $\beta_0 = 0$, a homogeneous conjugation problem with the same expression for the coefficient was formulated and solved in /12/. It was shown that when condition (4.5) was satisfied, its index was equal to zero. For this reason the inhomogeneous problem has a unique solution, and this makes it possible to write the pressure $p_2(x/\delta)$ at the surface of the profile in the neighbourhood of the trailing edge in the form of Fourier integrals. The resulting formulas are, however, very bulky, and are not given here.

The solution constructed is of interest in its own right but its chief importance is in determining the constant A_1 in (3.2) which enables us to close the external problem. This constant, proportional to the correction to the circulation is calculated by asymptotic joining of the expression for $p_2(x/\delta)$ for large values of the argument with two terms of the expansion of the pressure in the external region. The asymptotic behaviour of the functions defined by the Fourier integrals for large values of the argument is calculated by the stationary phase method. This requires a knowledge of the form of the kernel for small λ and simplifies the final formulae considerably.

The solution obtained does not satisfy the condition of adhesion at the boundary. An additional region of size $x = O(\delta)$, $y = O(\delta^{1+n/4})$ in which the equations of motion of the fluid have the structure of the Prandtl equations, offers the possibility of continuing the solution right up to the body. As we said before, a solution in this region has not been constructed, although its existence is assumed. When this assumption holds, the pressure distribution over the profile with a length dimension of the order of $O(\delta)$, differs from p_2 only in terms of the higher order of smallness.

5. Since, in order to satisfy the boundary conditions at the cut and at the corresponding line within the profile contour, resulting from inversion transformation, the sources and vortices were distributed along them, and the condition of impermeability at the surface was replaced by the first relation of (3.1), it follows that the expression for the pressure forces has a more complicated form than the Zhukovskii theorem would require. In particular, the presence of the sources at the profile contour and outside it results in the fact, that the projection of the pressure force on the direction of the incoming flow becomes non-zero. To a first approximation this projection is equal to zero (d'Alembert's paradox). Moreover, it should be remembered that in the neighbourhood of the trailing edge of the profile the flow parameters vary according to the inner solution. Therefore, in order to determine the pressure, we construct a composite, two-term expansion of the pressure at the surface of the profile.

We also note that completing the solution of the outer problem in the second approximation requires that the pressure at the surface of the body be recomputed, taking into account its drop across the boundary layer determined by the last equation of (3.1). In the case of a flow past an airfoil profile the correction is found to be vanishingly small everywhere outside the neighbourhood of the trailing edge. This is connected with the fact that the curvature of the profile outside the bow is small, as well as the displacement thickness and loss of momentum in the bow region where the boundary layer is very thin. Near the trailing edge the pressure is corrected within the composite expansion ifself.

6. Using the two term asymptotic expansion of the solution of the problem of incompressible fluid flow past a profile at high Reynolds numbers, we wrote a program for determining

its aerodynamic characteristics. The program includes a conformal transformation, computation of the boundary layer and wake, and corrections to the lift and general drag of the profile. The laminar boundary layer is computed from the point at which the flow fans out in the region of the leading edge using one of the versions of the method of integral Dorodnitsyn relations /13/, and the position of transition points on the upper and lower surface of the profile is determined using the empirical method from /14/. The position of these points may be fixed earlier. The integral method of /15/ is used to compute the turbulent boundary layer and wake. This is followed by determining the values of the functions $Q_1(\lambda)$ and $Q_2(\lambda)$ by solving problem (4.7), (4.8) by Euler's method. The velocity profile of the boundary layer is given in the form

$$U_0 = (1 - \cos \pi y_2)/2 + u^* (C + a/k) [y_2^{1/n} + (\cos \pi y_2 - 1)/2] \tag{6.1}$$

$$C = 5.1,\ k = 0.41,\ a = \ln (R_\delta u^*),\ n = a + Ck$$

Here R_δ is the Reynolds number calculated from the thickness of the boundary layer, u^* is the dynamic velocity referred to the velocity of flow at the outer boundary of the boundary layer, and the constants C and k are the same as in the formula for the velocity profile due to Coles /16/. Formula (6.1) is obtained by replacing, in Coles' expression for the velocity profile, the logarithmic function by the power function for $n \gg 1$. The number n is determined by the conditions at the boundaries of the boundary layer $U_0(1) = 1$, $U_0(0) = 0$.

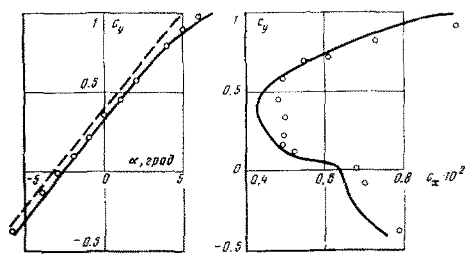

Fig.1

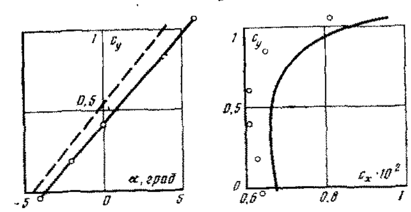

Fig.2

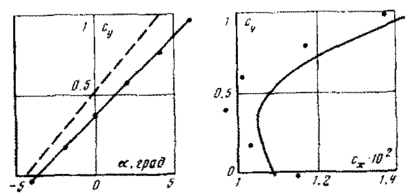

Fig.3

Since the expression for the velocity profile was obtained empirically, it follows that it can be approximated by another expression without violating any laws. It is interesting to note that the law of velocity written in the form /17/

$$U_0(y_2) = u^* C_1(n)\, y_2^{1/n} \exp (a/n)$$

yields, on comparing it with (6.1) as $y_2 \to 0$, the expression

$$C_1(n) = (n/k) \exp (Ck/n - 1)$$

which approximates the values given in the table in /7/ for various n, with 5% accuracy.

7. The theory constructed here of attached flow of a viscous fluid past a profile at high Reynolds numbers in the presence of a turbulent boundary layer, takes into account all terms of the order of $O(\delta^*)$. Certain assumptions which were made during its formulation and numerical realization, can only be validated by comparing the results obtained with experimental data. Such a comparison will also yield the relative accuracy of the mathematical model used. At

present, a fairly large number of computations have been carried out, and we show three of them in Figs.1-3 in the form of the relations $c_y(\alpha)$ and polars $c_y(c_x)$.

Fig.1 shows results for the profile NACA 64_2-415, with Reynolds number equal to $6\cdot10^6$. The profile has zero angle between the tangents to the trailing edge, a considerable region of laminar flow at the upper surface, and a loaded rear part. Fig.2 shows the characteristics of the profile NACA 4412 with the same Reynolds number. In this case the aperture angle at the trailing edge is non-zero, and its contribution towards the correction of the circulation amounts to 30% of its value. Moreover, the rearrangement of the flow at the trailing edge generates quite a large proportion of resistance due to pressure. Fig.3 shows the characteristics of the flows with a fixed transition point.

The points in Figs.1-3 correspond to experimental data /18/, and the dashed lines represent the values of the lift coefficient obtained without taking the viscosity into account. Comparing the results obtained we find that the error of the proposed theory does not exceed 2% when computing c_y, and does not exceed 10% when computing the drag coefficient. These values are comparable with the error of the experiment itself.

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